# Towards Computational Institutional Analysis: Discrete Simulation of a 3P Model 

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## Introduction

In the last decade, social scientists have shown growing interest in the formal analysis of social institutions. ${ }^{1}$ Economists, sociologists, political scientists and philosophers of science have contributed to this formal and mathematical modelling of institutions (their emergence, dynamic properties and stability).

At the same time computer simulations of social phenomena shifted from 'traditional' numerical simulations based on mathematical equations to agentbased, discrete event simulations. This new computational approach to modelling and simulating social phenomena has given birth to several new fields of research such as computational organization theory (Prietula and Carley, 1994), (Prietula, Carley and Gasser, 1998), computational sociology (Bainbridge et al., 1994), computational anthropology (Doran, 1995, Dean at el., 1998), computational social psychology (Nowak and Vallacher, 1998) and last but not least, computational economics (Tesfatsion, 1998).

The aim of this paper is to contribute to this new research agenda by adding computational institutional analysis or briefly CIA $^{2}$ to the list of computational social fields. As we envision it, CIA combines the formal modelling of social institutions with new methods of doing agent-based computational social science. The present paper is a step in this direction. We take up an economic, game theoretical model and investigate its potential for the understanding of institutions by means of simulation.

[^0]One way of theorizing about social order is by classifying actors in terms of the types of the actions they perform. Going back to a very basic, almost 'prehistoric' level, three broad types of of activities which seem to be promising for this task are production, predation and protection, where predation is understood to comprise all forms of taking away things or resources from a person against that person's will, and protection means to protect own's own possessions, resources and body. The proportions in which members engage in these activities may be used to draw distinctions between different forms of social organization - whether historical-empirical or merely conceptual. For example, in a society of slave holders (Knight, 1977) the slaves do not engage in protection, whereas the peasants in a peasant society do so. Conversely, the slave owners spend quite some effort on protection, much more than does a leader in a rural society. More precisely, the approach consists of looking at the proportions of time which a person devotes to production, predation and protection, to use these proportions for a classification of the persons, and to analyze the relative sizes of the classes to obtained. A person spending almost all her time on production thus may be classified as a producer, ${ }^{3}$ while it is not easy to find a natural label for, say, a person devoting her time equally to production, predation and protection eventhough the kind of such persons is determined theoretically in a precise way.

This approach may be pursued by starting from simple, 'institution-free' economic models in which the optimal or equilibrium distribution of persons' times is studied in a game theoretic setting. We here generalize a simple onegood, two-agent hobbesian model studied in (Houba \& Weikard, 1995) which deals with the optimal allocation of actors' times on the three kinds of activities: production, predation, and protection, this is why we speak of a 3P model. On the basis of his utility function which depends on the amounts of time all actors spend on each activity, each single actor tries to optimally distribute a fixed, total amount of time among the three types of activities. As game theoretic analysis becomes very difficult, if not practically impossible, for numbers of actors greater than two, simulation offers itself as the natural tool to be used.

We introduce the generalized 3P model, and describe how this is simulated in a discrete event setting. We then explore its potential by investigating the connections between actors' abilities to produce, predate and protect, the percentages in which these abilities are present in the population, and the times which actors spend on the three activities. The connections we found in different simulations are critically discussed in the light of corresponding, presystematic expectations. We describe some expected, 'nice' results, but also some unexpected results indicating deficiencies of the present, basic model. In spite of these negative results we believe that the model has a great potential for modifications and refinements.

[^1]A first positive result is that predation is 'robust' in the sense that actors who are best at predating (i.e. whose ability for predating exceeds that for production and protection) in most cases spend almost all their time on predation. Moreover, the time spent on predation increases sharply with an increase of the ability to predate, and does not much depend on variation of the other abilities. This finding points to a natural incentive which theoretically could back Hobbes' state of nature. A second positive result is that production time also increases with an increase of the ability to produce, though the degree of increase varies with other parameters, in particular with the coefficients for the other abilities and the percentage of producers in the population. This also indicates a natural incentive, and the variability of increase opens the way for studying the systematic effects of other, 'external' parameters on the incentive to produce.

Negatively, we found that protection time in most cases does not monotonically increase with protection ability. A first interpretation is that the ability for protection is dominated by the other two abilities, and thus not really an independent variable. This interpretation is also supported by the intuitive observation that the abilities for predation and protection in a pre-historic environment are closely related to similar kinds of bodily skills and strengths.

A second negative result is that in simulations where abilities were lognormally distributed in the population, we were not able to produce patterns of time proportions corresponding to presystematic, real-life expectations. For example, a 'real-life' state would obtain when there are $66 \%$ of producers and $34 \%$ of predators such that producers spend all their time on production and predators split their time on predating and protection. Even a systematic search algorithm could not find ability coefficients which through simulation would produce such distributions. This indicates that the form of the utility functions used in the model is still too rigid and restricted.

## 1 The Basic Hobbesian 3P Model

It may seem strange to start an analysis of social institutions by modelling an institution-free hobbesian world. But as noted by (Wolff, 1996) in his analysis of Hobbes' state of nature, "To understand why we have something, it is often a good tactic to consider its absence". Hence, one way to examine how social institutions emerge and what type of social interactions (exchange based vs. power based) underlie these institutions, is to start from an institution-free setting of which Hobbes' account is perhaps the most famous example.

Since Bush's pioneering work (Bush, 1976) there have been numerous articles and books ${ }^{4}$ devoted to the modelling of conflictual anarchy of the hobbesian variety. ${ }^{5}$ We here study a simple representative of the hobbesian variety of mod-

[^2]els in order to show how such a model, when generalized to a multi-agent computational world, may give rise to interesting features that could (practically) not be found by paper and pencil. However since, as pointed out by (Binmore, 1998), computer simulations are not a substitute for deductive reasoning based on sound theoretical microeconomics or game theory we shall first give a brief account of the theoretical model underlying our multi-agent simulations.

In the hobbesian world, there are no property rights or social norms to regulate agent interactions. In order to survive in such a world, individual agents undertake three basic types of activities: they produce, they use force to steal (predate) and they protect themselves against the predatory activities of others.

People are not equal in their abilities for doing so. Some are stronger than others, some are better at producing than at stealing. Depending on their relative abilities individuals produce, steal and protect themselves by equating the marginal returns of these three basic activities. The results of an actor's marginal calculus depend on the behavior of the other agents with whom she interacts. In most approaches this interactive behavior is modelled by CournotNash type assumptions but a few models use Stackelberg type (leader-follower) assumptions.

Adopting the two-persons generalization of (Houba and Weikard, 1995) of Bush's original model, let us consider two persons $i, j$. Let $P_{i 1}, P_{i 2}, P_{i 3}$ be the production, predation and protection functions of individual $i$ (those of $j$ are obtained by interchanging $i$ and $j)$. $P_{i p}$ specifies the utility which person $i$ derives from production ( $p=1$ ), predation ( $p=2$ ) and protection ( $p=3$ ). There is only one good which is produced, and taken away by predators.
(1) Production: $P_{i 1}=f_{1}\left(a_{i 1}, t_{i 1}\right)$
(2) Predation: $P_{i 2}=f_{2}\left(a_{i 2}, t_{i 2}, P_{j 1}, P_{j 3}\right), i \neq j$
(3) Protection: $P_{i 3}=f_{3}\left(a_{i 3}, t_{i 3}\right)$,
where the $a_{i p}>0$ are individual parameters for, respectively, the productive ( $p=1$ ), predatory ( $p=2$ ) and protective ( $p=3$ ) capacities of individual $i$, called abilitiy coefficients in the following, and $t_{i p}$ denotes the time devoted by individual $i$ to activity number $p$. Whereas the production and protection functions (1 and 3) depend only on $i$ 's own parameters and variables, the predation function (2) includes arguments that do not only depend on $i$ 's own capacities and time devoted to predation. The predation function also depends on the other person's time and capacities devoted to production and protection. The more

[^3]$j$ produces the more $i$ can steal from him, but the more $j$ protects himself the more costly is it to steal from him.

Each individual $k$ has an overall utility function $U_{k}$ it seeks to maximize. A simple form for $U_{i}$ suggested by (Houba \& Weikard, 1995) is this:
(4) $U_{i}=U_{i}\left(t_{i 1}, t_{i 2}, t_{i 3}, t_{j 1}, t_{j 2}, t_{j 3}\right)=P_{i 1}+P_{i 2}-P_{j 2}, j \neq i$

Thus $U_{i}$ is equal to what $i$ gets out of producing (captured by $P_{i 1}$ ) plus what she gets out of stealing from $j$ (captured by $P_{i 2}$ ) minus what $j$ gets out of predating on $i$ (captured by $P_{j 2}$ ).

In (Houba \& Weikard, 1995) the functions $f_{1}, f_{2}$ and $f_{3}$ are generally specified as follows. For $k=i, j$,
(7) $f_{k 1}\left(a_{k 1}, t_{k 1}\right)=a_{k 1} t_{k 1}$ and $f_{k 3}\left(a_{k 3}, t_{k 3}\right)=a_{k 3} t_{k 3}$
$f_{i 2}\left(a_{i 2}, t_{i 2}, P_{j 1}, P_{j 3}\right)=a_{i 2}\left(t_{i 2}\right)^{\alpha_{i}} P_{j 1}\left(1-P_{j 3}\right)$, and $f_{j 2}\left(a_{j 2}, t_{j 2}, P_{i 1}, P_{i 3}\right)=a_{j 2}\left(t_{j 2}\right)^{\alpha_{j}} P_{i 1}\left(1-P_{i 3}\right)$.

Using (7) we obtain the following general expressions for $U_{i}$ and $U_{j}$.

$$
\begin{aligned}
& \text { (8) } U_{i}\left(t_{i 1}, t_{i 2}, t_{i 3}, t_{j 1}, t_{j 2}, t_{j 3}\right)=a_{i 1} t_{i 1}+a_{i 2}\left(t_{i 2}\right)^{\alpha_{i}} a_{j 1} t_{j 1}\left(1-a_{j 3} t_{j 3}\right)- \\
& a_{j 2}\left(t_{j 2}\right)^{\alpha_{j}} a_{i 1} t_{i 1}\left(1-a_{i 3} t_{i 3}\right), \\
& U_{j}\left(t_{j 1}, t_{j 2}, t_{j 3}, t_{i 1}, t_{i 2}, t_{i 3}\right)=a_{j 1} t_{j 1}+a_{j 2}\left(t_{j 2}\right)^{\alpha_{j}} a_{i 1} t_{i 1}\left(1-a_{i 3} t_{i 3}\right)- \\
& a_{i 2}\left(t_{i 2}\right)^{\alpha_{i}} a_{j 1} t_{j 1}\left(1-a_{j 3} t_{j 3}\right) .
\end{aligned}
$$

Each actor $k$ seeks to maximize his utility subject to the constraint that $t_{k 1}+$ $t_{k 2}+t_{k 3} \leq T$ where $T$ is the total amount of time available in the period considered which, for reasons of simplicity, is set equal to 1 for both actors. In other words, each actor $k$ tries to find an optimal allocation of times $\left(t_{k 1}, t_{k 2}, t_{k 3}\right)$. Clearly, both actors are strategically interdependent since in (8) $i$ 's utility depends on the times chosen by $j$ and conversely. The resulting game can be analytically solved for two actors.

## 2 The General Model

We generalize this model to the case of $n$ actors as follows, retaining the assumption of one single good that is produced by everyone. Each of the $n$ actors $i(i=1, \ldots, n)$ has a utility function $U_{i}$ depending on the $3 n$ times which all actors spend on the three activities: production, predation and protection. For each $i$, the times $i$ spends on production, predation and protection, respectively, are denoted by $t_{i}^{1}, t_{i}^{2}$ and $t_{i}^{3}$. Thus $i$ 's distribution of time on the three activities is given by $\overrightarrow{t_{i}}=\left(t_{i}^{1}, t_{i}^{2}, t_{i}^{3}\right)$ and $i$ 's utility function may be written as $U_{i}=U_{i}\left(\overrightarrow{t_{1}}, \ldots, \overrightarrow{t_{n}}\right)$. When the time distributions of the other actors $j, j \neq i$, are held constant, we simply write $U_{i}=U_{i}\left(\overrightarrow{t_{i}}\right)$. We assume that i's utility function has the following form
(9) $U_{i}\left(\overrightarrow{t_{1}}, \ldots, \overrightarrow{t_{n}}\right)=a_{i 1} t_{i 1}$

$$
\begin{aligned}
& +a_{i 2}\left(t_{i 2} /(n-1)\right)^{\alpha_{i}} \Sigma_{j}\left(a_{j 1} t_{j 1}\left(1-a_{j 3} t_{j 3}\right)\right) \\
& -\min \left(1,\left(\Sigma_{j} a_{j 2}\left(t_{j 2} /(n-1)\right)^{\alpha_{j}}\right)\right) a_{i 1} t_{i 1}\left(1-a_{i 3} t_{i 3}\right)
\end{aligned}
$$

where $0<\alpha_{i}<1,0 \leq a_{i 1}, a_{i 2}, a_{i 3}$ and $a_{i 1}+a_{i 2}+a_{i 3}=1$ for $i=1, \ldots, n$. The ability coefficient $a_{i p}$ expresses the 'ability' or 'efficiency' with which actor $i$ performs activity number $p(p=1,2,3$ for production, predation, protection), and $t_{i p}$ is the time $i$ spends on activity $p$. The three components of $U_{i}$ in (9) may be interpreted as follows. The first component $a_{i 1} t_{i 1}$ represents the amount of the single good which $i$ produces (or the utility she derives from this amount), depending on her productive ability $a_{i 1}$ and the time $t_{i 1}$ she spent on production.

The second component may be best understood if we rewrite it as $(n-$ 1) $\left[a_{i 2}\left(t_{i 2} /(n-1)\right)^{\alpha_{i}}(1 /(n-1)) \Sigma_{j} a_{j 1} t_{j 1}\left(1-a_{j 3} t_{j 3}\right)\right] . a_{i 2}\left(t_{i 2} /(n-1)\right)^{\alpha_{i}}$ is the 'weight' of $i$ 's activity of predating when $i$ predates one of the $n$ other actors, on the assumption that $i$ splits his 'predation time' equally on all other actors. The average, 'non-protected' production of some actor thus predated by $i$ is $(1 /(n-$ 1)) $\Sigma_{j} a_{j 1} t_{j 1}\left(1-a_{j 3} t_{j 3}\right)$. So $a_{i 2}\left(t_{i 2} /(n-1)\right)^{\alpha_{i}}(1 /(n-1)) \Sigma_{j} a_{j 1} t_{j 1}\left(1-a_{j 3} t_{j 3}\right)$ is $i$ 's utility from predating one 'average' fellow actor. In order to obtain $i$ 's total utility this expression has to be taken $n-1$ times.

In the third part, $\left(t_{j 2} /(n-1)\right)^{\alpha_{j}}$ gives the 'size' or 'weight' of that part which $j$ can take away from $i$ 's non-protected product $a_{i 1} t_{i 1}\left(1-a_{i 3} t_{i 3}\right)$ on the assumption that $j$ spends her 'predation time' $t_{j 2}$ equally on all other actors. Thus the third part refers to the sum of all parts which are taken away from $i$ 's non-protected product by all the other actors. Since in the case of more than two actors the sum of all 'weights' may be greater than 1 we have to take the minimum of this sum and 1 in order to prevent a change of sign in the third component.

As an analytic treatment of these general equations is very difficult, if not practically impossible, the best way to proceed is by simulation. We use a discrete event simulation shell called SMASS (Sequential Multi-Agent System for Social Simulation) written in SWI-PROLOG (Balzer, 1999), (Wielemaker, 1993,1996). This shell executes simulation runs over a fixed number $N$ of periods such that in each period, each actor is called up for action exactly once. The task of implementation in this shell reduces to the formulation and implementation of a rule of behavior according to which each actor acts when called up in a period $T$.

## 3 The Simulation

As the above analytic model is static, we have to find a way using a dynamical simulation in order to obtain the static distributions of actors' times devoted to the three different activities. This is done as follows. The model's total time interval which is captured in one simulation run, is represented by the number $N$ of all periods over which the simulation is run. Assuming that each actor in
each period acts just once we can count the numbers $m_{1}, m_{2}, m_{3}$ of periods in which he produces, predates, or engages in protection, so $N=m_{1}+m_{2}+m_{3}$. We identify these numbers $m_{1}, m_{2}, m_{3}$ with the times $t_{1}, t_{2}, t_{3}$ an actor spends on the three activities in the solution of the analytical model.

A second problem is to formulate a rule of behavior expressing the maximization assumptions which in the analytic model are applied to the equations (1)-(4) and (5) and (6) above. In principle, one could try to just let each actor solve the above equations and distribute her time according to that solution. This is impractical, however, because we consider more than two actors, and for larger numbers we simply wouldn't know how to solve the equations. We therefore formulate a different basic rule of behavior as a substitute for the assumptions of the analytic model.

To this end during the course of the simulation a 'history' is built up recording in each period $T$ the numbers of periods every single actor spent on each of the three activities up to the present period $T$. Thus if actor $i$ is called up in period $T$ her history $\overrightarrow{h_{i, T}}$ will consist of three numbers $\overrightarrow{h_{i, T}}=\left(h_{i 1, T}, h_{i 2, T}, h_{i 3, T}\right)$ such that $h_{i 1, T}+h_{i 2, T}+h_{i 3, T}=T$ and each $h_{i p, T}$ is the nmuber of periods in which $i$ performed activity number $p(p=1,2,3)$. Such a history gives the distribution of the times $i$ spent on the three activities.

Instead of the utilities $U_{i}\left(\overrightarrow{t_{1}}, \ldots, \overrightarrow{t_{n}}\right)$ derived from the 'final' proportions of times we now may consider utilities derived from the relative proportions of times spent up to a given period $T$, i.e. utilities depending on the actors' histories up to $T$
$U_{i}(T)=U_{i}\left(\overrightarrow{h_{1, T}}, \ldots, \overrightarrow{h_{n, T}}\right)$, where $\overrightarrow{h_{i, T}}=\left(h_{i, T}^{1}, h_{i, T}^{2}, h_{i, T}^{3}\right)$
We apply the following rule of behavior. An actor $i$ in period $T$ calculates the marginal utilities for each of the three activities, and chooses that activity which yields highest marginal utility. The marginal utilities are those which actor $i$ would derive from spending one more period on production, predation or protection, given that up to period $T$ he spent the times $\left(h_{i, T}^{1}, h_{i, T}^{2}, h_{i, T}^{3}\right)$ on these activities. $i$ 's marginal utility for production in period $T$ is thus defined by
(10) $U_{i}\left(\overrightarrow{h_{1, T}}, \ldots,\left(h_{i, T}^{1}+1, h_{i, T}^{2}, h_{i, T}^{3}\right), \ldots, \overrightarrow{h_{n, T}}\right)-U_{i}\left(\overrightarrow{h_{1, T}}, \ldots, \overrightarrow{h_{n, T}}\right)$.

The marginal utilities for predation and protection are obtained in the same way by adding in (10) one period to $h_{i, T}^{2}$ and $h_{i, T}^{3}$, respectively. ${ }^{6}$

In (10) the other actors' histories enter in the calculation of $i$ 's marginal utilities; these are taken as they are found at the time of execution in period $T .{ }^{7}$

In the analytical model a solution or state of equilibrium is a list of time distributions $\left(\overrightarrow{t_{1}}, \ldots, \overrightarrow{t_{n}}\right.$ ) (a 'state') satisfying a condition of maximality or equi-

[^4]librium. In the simulation such a state corresponds to the actors' final histories $\left(\overrightarrow{h_{1, N}}, \ldots, \overrightarrow{h_{n, N}}\right)$ where $N$ denotes the total number of periods for which the simulation is run. While the simulation is running, the histories $\overrightarrow{h_{i, T}}$ steadily change when $T$ grows from 1 to $N$. However, we can say that the system in state $\left(\overrightarrow{h_{1, T}}, \ldots, h_{n, T}\right)$ has become stable if the fractions $h_{i p, T^{\prime}} / T^{\prime}$ do not change significantly for all $T^{\prime}$ such that $T \leq T^{\prime}$. For instance, when the final distribution of $i$ 's time is $(0.5,0.5,0)$ - i.e. $i$ spent half of her time on producing and half of it on predating - then for $N=100, \overrightarrow{h_{i, N}}=(50 / 100,50 / 100,0)$. When the system has become stable, say in period 70 , then $\overrightarrow{h_{i, 70}}=(35 / 70,35 / 70,0)$ and these fractions will show only insignificant deviations for $T>70$. As the system operates with integers, they cannot remain strictly identical because, say, for $N=100$, in each period one of the history's components will be increased by $1 / 100$.

The states which are stable in this sense may be taken as the analogues of analytic solutions. All simulations were run for 100 periods, and a stable state was reached when deviations were allowed up to $\epsilon=0.02$. The stable state in most cases was reached between periods number 60 and 80 .

## 4 Simulation Results

We performed a number of simulations in order to explore the space of possibilities given by variations in the parameters: numbers of actors, ability coefficients, exponents, and initial distributions of predators and producers in the population. This is a huge space and it does not seem a good idea to try to explore it fully systematically. We varied several items in more systematic fashion, but only so within relatively narrow boundaries. Each simulation was repeated ten times with the same initial data. The results reported here are the mean values over these repetitions, deviations from these means were usually in the order of 0.01 or less.

Even within a homogenous population of completely identical actors, slightly different results are observed for different actors. This effect is due to the multiagent character of the simulation in which it makes a difference, for instance, whether in a period one of the few predators is called up at the beginning or towards the end of the period, i.e. before most other actors have chosen their activities and acted, or after that. However, these individual differences usually are not significant, deviations being smaller than 0.02 , and usually much smaller. For this reason, we do not differentiate in the following description between single actors, and just report the results for one arbitrary, representative member of each sub-population. ${ }^{8}$

[^5]In a first series of simulations we used ability coefficients that are lognormally distributed in the population. As these coefficients consist of three components whose interdependency is difficult to judge empirically we used a mix of two different random processes to create them. We first created lognormally distributed numbers $b_{i}$ - one for each actor $i$ - within the interval $[0,1]$. We then split the 'rest' $1-b_{i}(\geq 0)$ randomly into two parts $b_{i}=a_{i}+c_{i}$, and used $\left(a_{i}, b_{i}, c_{i}\right)$ as coefficients of actor $i$.

Defining 'producers' $i$ as those actors whose ability for producing, $a_{2}^{i}$, is strictly greater than that for predating, $a_{2}^{i}$, and calling all other actors 'predators', the population split up into $\mathrm{x} \%$ of producers and (1-x) \% of predators. With varying numbers of actors x varied in the interval [40, 60].

The means of the ability coefficients and the time profiles did vary with variations of the number of actors, but this effect is mainly due to the fact that for a different number of actors, the ability coefficients are newly created in the random way described earlier. Table 1 summarizes some results.

Table 1

| number of actors | 10 | 50 | 100 |
| :--- | :--- | :--- | :--- |
| percentage of | 50 | 42 | 43 |
| producers | $\cdot$ | $\cdot$ | $\cdot$ |
| mean producer: | . | . | . |
| ability coeffs | $(.36, .10, .53)$ | $(.51, .14, .33)$ | $(.46, .13, .40)$ |
| time profiles | $(.55, .17, .27)$ | $(.23, .57, .18)$ | $(.18, .64, .17)$ |
| variances | $(.007, .041, .021)$ | $(.035, .174, .052)$ | $(.038, .180, .060)$ |
| of time | $\cdot$ | $\cdot$ | $\cdot$ |
| mean predator: | $\cdot$ | . | . |
| ability coeffs | $(.18, .58, .22)$ | $(.16, .59, .23)$ | $(.14, .60, .25)$ |
| time profiles | $(.13, .86,0)$ | $(.01, .98,0)$ | $(.01, .98,0)$ |
| variances | $(.009, .009,0)$ | $(0,0,0)$ | $(0,0,0)$ |
| of time | $\cdot$ | . | . |

Remarkably, in populations of more than 40 actors, a 'mean producer' spends more time on predating than on producing. This means that several single producers, i.e. actors who are more able to produce than to predate, nevertheless spend more time on predation, which, for them, is the inferior activity. This result is prima facie at odds with the assumption of rationality underlying the model. However, we can interpret it as showing that the incentive for predation which is incorporated in the form of the utility function is much stronger than that for production so that it surpasses the prima facie incentive given in terms of the ability coefficients.

By contrast, the 'mean predators' do not spend much time on producing eventhough they have a non-negligable coefficient for production. This confirms

[^6]the previous interpretation. Moreover, the 'mean predators' do hardly spend any time on protection, which in many simulation means that no single predator does so. This outcome conflicts with the intuition - external to the model - that predators also should predate on their 'fellow' predators. Given the high percentage of predators in the population (often more than $50 \%$ ), one would want to see a substantial amount of time spent by predators on protecting themselves against each other. However, this incentive is not expressed in the model. The third, negative part of the utility function in (9) depends multiplicatively on the actor's own product ( $a_{i 1} t_{i 1}$ ) which is negligible for predators. According to (9) a predator spending no time on production has nothing to protect. In reality, even in the basic case in which all products - whether produced or robbed are consumed in the same period, there is the possibility of one predator taking away from another one the good which the latter just robbed from a third person.

Looking at how each single time component $t_{r}$ (e.g. the time spent on predation, $r=2$ ) depends on a single ability coefficient $a_{s}$ (e.g. the coefficient for production, $s=1$ ), we arrange the coefficients that are present in the population in an increasing order so that for the set $\left\{i_{1}, \ldots, i_{n}\right\}$ of actors we obtain a series $a_{s}\left(i_{1}\right)<\ldots<a_{s}\left(i_{n}\right)$. When we plot the corresponding time $t_{s}\left(i_{j}\right)$ against each such coefficient $a_{s}\left(i_{j}\right)$ the dependence (in a population of 30 actors) can be graphically depicted as in figure 1.

Figure 1
a)
time spent on producing

ability coefficients for production
b)
time spent on predating

ability coefficients for predation
c)
time spent on predating

ability coefficients for protection

Distinguishing increase ( + ) from decrease ( - ), and degrees of the strengths of
the connections ( $1=$ strong and regular, $2=$ weak and regular, $3=$ irregular $)$ all the dependencies are summarized in table 2.

Table 2 increasing coefficient for

| time spent on | production | predation | protection |
| :--- | ---: | ---: | ---: |
| production | .+ | . | .$\dot{1}$ |
| predation | ,+ 2 | ,- 1 | ,+ 3 |
| protection | ,- 2 | ,+ 1 | ,- 3 |
| t, | ,+ 3 | ,- 1 | ,+ 3 |

These connections do not change when they are restricted to the two subpopulations of producers and predators.

The absence of a regular increase of protection time with an increase of protection ability (even in the subpopulation of producers) we find unsatisfactory. As producers' product increases over time (in the simulation), and as there are many predators, producers should have a strong incentive for protection which is also in accordance with the form of the utility function (9).

In these simulations on might suspect that the results depend on the initial creation of lognormally distributed ability coefficients. In order to control for this we conducted a second series in which we focused on one ability coefficient. When this was fixed, the percentages of producers, predators and protectors (defined in terms of abilities), as well as all the other coefficients were varied randomly. The random creation of the 'other' parameters was repeated 20 and 50 times. Doing the simulation for different values of the focused ability coefficient, like $0.2,0.25,0.3, \ldots, 1$, and plotting the times spent on one activity against the focused coefficient, we obtained qualitatively the same results as in the first series. Figure 2 shows some dependencies for the series $0.2,0.25,0.3$, $\ldots, 1$ of coefficients number $s$ on the x-axis and times number $r$ on the y -axis.
Figure 2
a)
time spent on predating
$\bigcirc 0000000000$ ○○○○○。
0
$\qquad$
ability coefficients for protection
b)
time spent on production

ability coefficients for production
c)
time spent on predation

ability coefficients for predation
In a third series, we investigated the sensitivity of the model in dependence of the absolute numerical values of the ability coefficients. Instead of normalized ability coefficients (adding up to 1 ) we used larger numbers, and studied the system's behavior for different, fixed sets of coefficients and proportions of producers and predators. We started with normalized coefficients, multiplied them by $10,20,30$ and gauged the ( $1-\ldots$ ) expressions in (9) to the absolute values, e.g. when using coefficients adding up to 10 , the ' 1 ' was replaced by ' 10 '. In a population of 20 actors we ran all combinations of coefficients ( $0.8,0,0.2$ ), $(0.4,0.4,0.2),(0.1,0.1,0.8)$ for producers, $(0,1,0),(0,0.7,0.3),(0.3,0.4,0.3)$ for predators and percentages $100,80,60,40,20$ of producers in the population.

There was no significant variation of predators' times in dependence on the absolute sizes of ability coefficients, and variation for producers was relatively
small, never exceeding $30 \%$. We may say that the model is moderately robust with respect to the absolute sizes of ability coefficients.

We also varied the exponents $\alpha_{i}$ attached to predation times in equation (9). In the earlier simulations these exponents had been uniformly set equal to $1 / 2$. In a fourth series the exponents $1 / 2$ were replaced by smaller and larger values ( $0.2,0.4,0.8,1$ ), but still each actor's utility function was calculated with the same exponents. Running the simulation in the setting of series 3 above we found that the times of predators are hardly affected by changes of the exponents. The main effect observed for producers was that when their percentage in the population decreases below a threshold, they split their times nearly equally on production and predation. The only effect of varying exponents is that this threshold decreases with growing exponent, but also with decreasing predating ability of the predators. In Table 3 we show some results. In the entries $x / y, x$ denotes the production time and y the predation time of a producer.

Table 3

| . | exponents | . | . |  |
| :--- | :--- | :--- | :--- | :--- |
| . | 0.2 | 0.4 | 0.8 | 1 |
| ability coeffs | $\cdot$ | . | . | . |
| producer $(.8,0, .2)$ | . | . | . | . |
| predator $(0,1,0)$ | . | . | . | . |
| percentage of | . | . | . | . |
| producers | . | . | . | . |
| $100 \%$ | $1 / 0$ | $1 / 0$ | $1 / 0$ | $1 / 0$ |
| $80 \%$ | $.5 / .5$ | $.5 / .5$ | $1 / 0$ | $1 / 0$ |
| $60 \%$ | $.5 / .5$ | $.5 / .5$ | $1 / 0$ | $1 / 0$ |
| $40 \%$ | $.5 / .5$ | $.5 / .5$ | $.7 / .3$ | $1 / 0$ |
| $20 \%$ | $.5 / .5$ | $.5 / .5$ | $.5 / .5$ | $.9 / .1$ |
| ability coeffs | . | . | . | . |
| producer $(.8,0, .2)$ | . | . | . | . |
| predator $(.3, .4, .3)$ | . | . | . | . |
| percentage of | . | . | . | . |
| producers | . | . | . | . |
| $100 \%$ | $1 / 0$ | $1 / 0$ | $1 / 0$ | $1 / 0$ |
| $80 \%$ | $.8 / .2$ | $1 / 0$ | $1 / 0$ | $1 / 0$ |
| $60 \%$ | $.5 / .5$ | $.8 / .2$ | $1 / 0$ | $1 / 0$ |
| $40 \%$ | $.5 / .5$ | $.5 / .5$ | $1 / 0$ | $1 / 0$ |
| $20 \%$ | $.5 / .5$ | $.5 / .5$ | $1 / 0$ | $1 / 0$ |

In a final series we tried to reproduce 'reasonable' empirical time distributions as found in existing populations. For example, in a slave holder society, a first guess for time distributions would be $(1,0,0)$ for slaves (which form, say 40 percent of the population), and $(0,0.6,0.4)$ for non-slaves (making up 60 percent of the population). That is, slaves spend all their time on production, while
non-slaves split their time on $60 \%$ of predation and $40 \%$ of protection. We started a search program which tried to find ability coefficients for which the time distributions resulting in a simulation with such coefficients fitted with the times and percentages fixed beforehand.

This resulted in complete failure. For none of three 'reasonable', initial time distributions and percentages the program found coefficients such that the simulation results would fit with the given times and percentages. Even if we admit that the search algorithm used is perhaps very inefficient this indicates that the model in its present form is not sufficiently flexible.

## Conclusion

First simulations with a multi-agent model in which actors optimize the time distributions for production, predation and protection yield insight in the rational, non-institutionalized incentives for engaging in each of these activities. On the one hand, predation and production times increase with the actors' respective abilities which points to a basic, natural incentive to engage in production and predation. On the other hand, protection time does not systematically vary with either of the three ability coefficients which indicates that perhaps it is not an independent variable.

We were not able with the present model to produce 'real life' time distributions and percentages of producers and predators. This may have two reasons. First, the model's basic equation (9) may be too rigid or too restricted. In future research we will use variations of the model with different exponents and different overall forms of (9) to find 'solutions' which reproduce given, plausible time distributions and percentages. In particular, the absence of predation among predators in (9) has to be removed.

A second reason for failure may be the neglect of institutional features. Broadly speaking, institutions seem to produce and to stabilize certain patterns of time distributions and percentages which do not naturally occur in an institution-free state. We hypothesize that the present model allows to incorporate some such institutional features, which we hope to find and to inclcude in the picture.

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[^0]:    ${ }^{1}$ The formal approach based on deductive reasoning is sometimes opposed to the descriptive approach of the 'old' institutionalist school of the Commons variety (Commons, 1934). However, such an opposition between a theoretically driven 'new' institutionalism and an 'anti-theoretic' old institutionalism does not seem adequate; see (Hodgson, 1998) who stresses the early institutionalists' concern for theoretical issues.
    ${ }^{2}$ This field of research has something in common with its more famous counterpart. Computational Institutional Analysis is at the center of economic analysis, it is (artificial) intelligence based and, finally, it is agent based. Needless to say, we do not pursue the same objectives.

[^1]:    ${ }^{3}$ In the present paper we will use another definition of producers, namely in terms of their abilities to produce.

[^2]:    ${ }^{4}$ A critical review of these models is found in (Albert, 1999).
    ${ }^{5}$ We are reluctant to use the term 'anarchy' in connection with conflictual models opposing

[^3]:    bandits (predators) to peasants (producers) because this tends to confirm the widespread prejudice that anarchy implies fighting or a hobbesian state of nature. Originally, anarchy only means absence of domination. Though predation, robbery and exploitation are compatible with the absence of domination, they are by no means implied by such absence, as Hobbes made us believe. See (Flap, 1985) for a counter example. The hobbesian state of nature in which everyone fights everyone is only one among many other conceptual - including less frightening - alternatives. See also the comments of (Dowd, 1997) on Hirshleifer's model (Hirshleifer, 1995) of conflictual anarchy.

[^4]:    ${ }^{6}$ As periods are represented by integers, the natural unit here is 1 .
    ${ }^{7}$ This means asynchronous updating.

[^5]:    ${ }^{8}$ As a warming up exercise we simulated the Houba-Weikard 2-actor model with the coefficients $[2,0,1]$ for the producer and $[1,1,0]$ for the predator. This yields the expected Nash equilibrium at $(0,0.3968,0.2063)$ - the remaining times being uniquely determined by the time

[^6]:    constraint - for the predator, which in this case also can easily be computed by hand.

